

1.1 – Parent Functions and Transformations

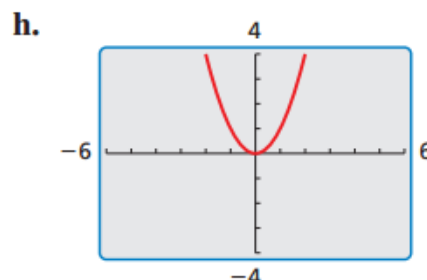
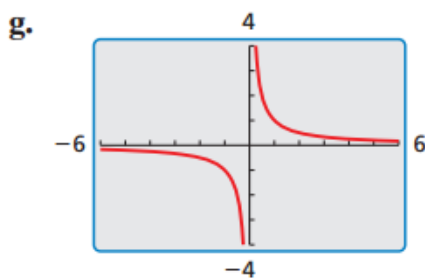
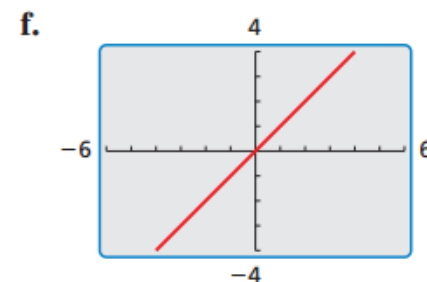
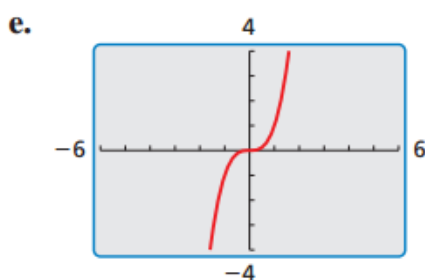
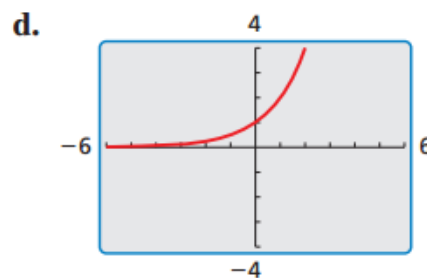
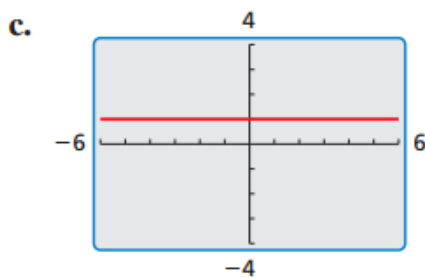
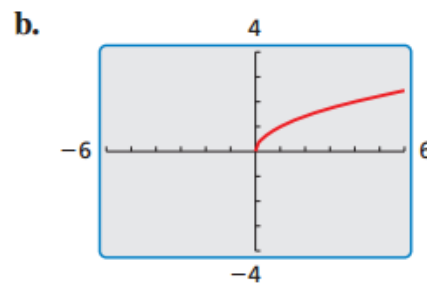
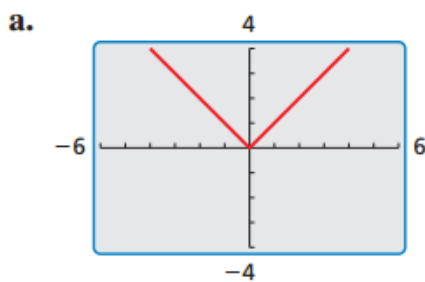
Essential Question: What are the characteristics of some of the basic parent functions?

What You Will Learn

- Identify families of functions.
 - Describe transformations of parent functions.
 - Describe combinations of transformations.
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Identifying Basic Parent Functions

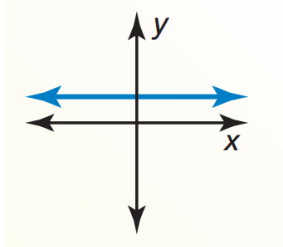
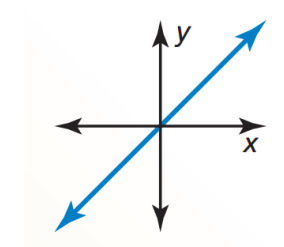
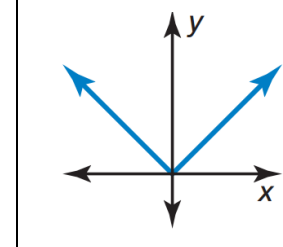
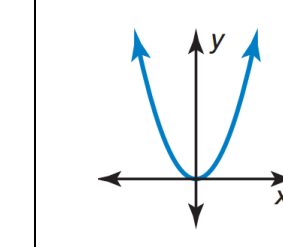
Graphs of eight basic parent functions are shown below. Classify each function as:
constant; linear; absolute value; quadratic; square root, cubic, reciprocal; or exponential



Identifying Function Families

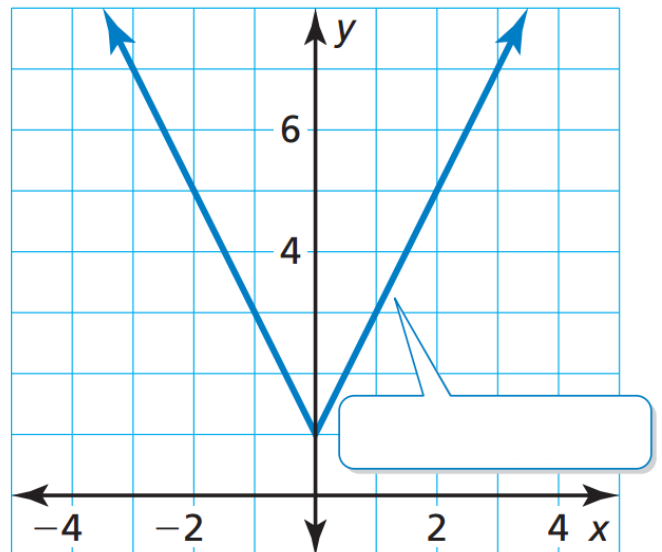
Functions that belong to the same **family** share key characteristics. The _____ is the most basic function in a family. Functions in the same family are **transformations** of their parent function.

Core Concept – Parent Functions

Family				
Rule				
Graph				
Domain				
Range				

Example 1: Identifying a Function Family

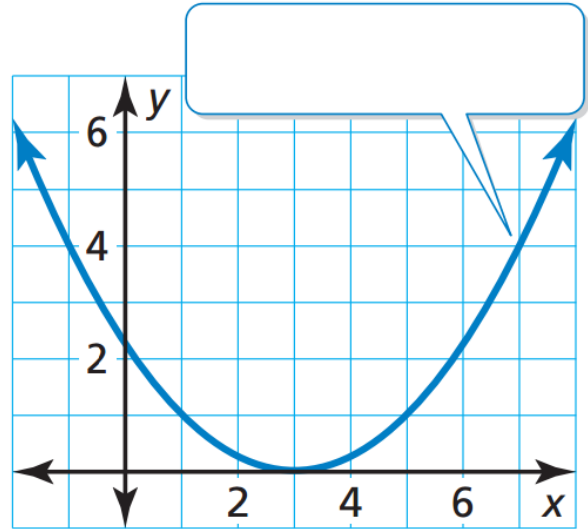
Identify the function family to which f belongs. Compare the graph of f to the graph of its parent function.



Within a family, members may think or look alike, but they often have some differences. Function “families” work the same way. Graphs in a function family have traits that look the same, but they also have some minor differences.

Example 2:

Identify the function family to which g belongs. Compare the graph of g to the graph of its parent function.



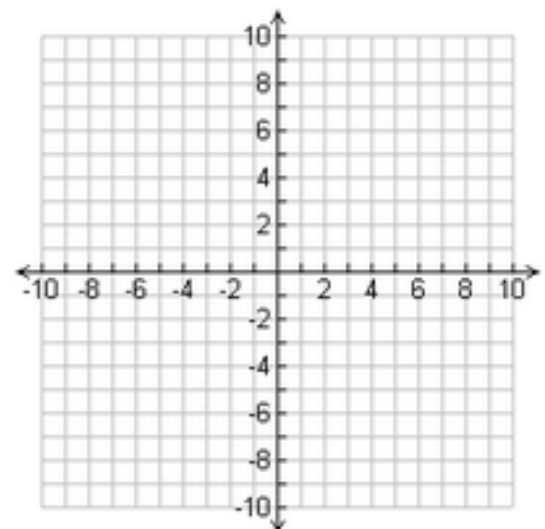
Describing Transformations

A _____ changes the size, shape, position, or orientation of a graph.

A _____ is a transformations the shifts a graph horizontally, and/or vertically but does not change its size, shape, or orientation.

Example 3:

Graph $g(x) = x - 4$ and its parent function. Then describe the transformation.

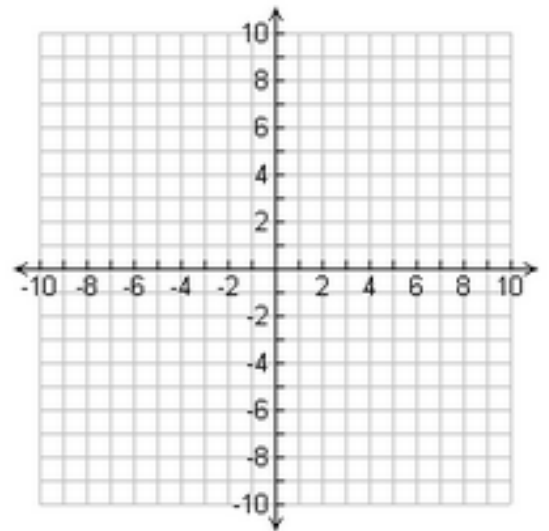


A _____ is a transformation that flips a graph over a line called the **line of reflection**. A reflected point is the same distance from the line of reflection as the original point but on the opposite side of the line.

Graphing and Describing Reflections

Example 4:

Graph $p(x) = -x^2$ and its parent function. Then describe the transformation.

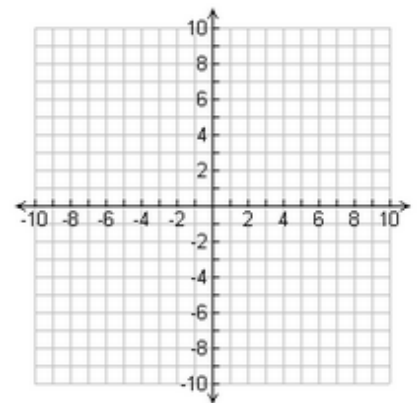
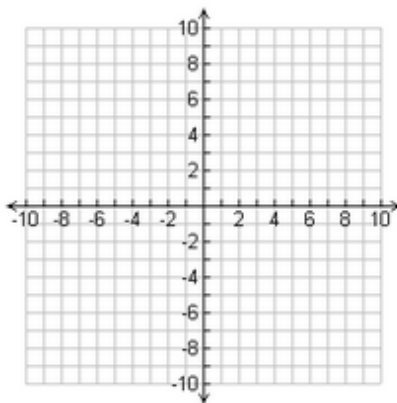
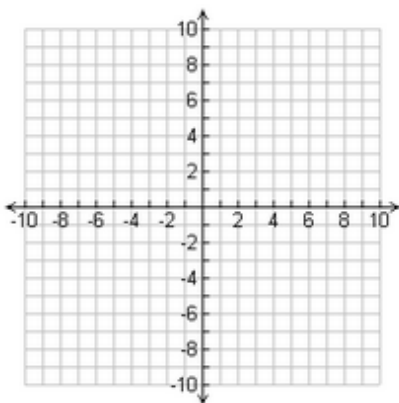


Example 5: Graph the function and its parent function. Then describe the transformation.

(a) $g(x) = \sqrt{x} + 3$

(b) $h(x) = (x - 2)^2$

(c) $n(x) = -|x|$



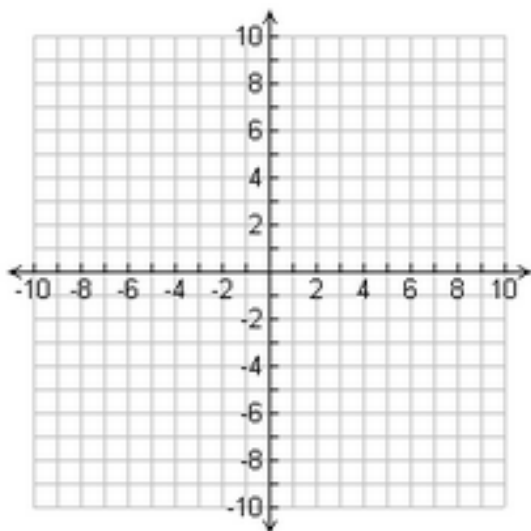
Another way to transform the graph of a function is to multiply all of the y-coordinates by the same positive factor (other than 1). What the factor is greater than 1, the transformation is a _____ . When the factor is greater than 0 and less than 1, the transformation is a _____ .

Graphing and Describing Stretches and Shrinks

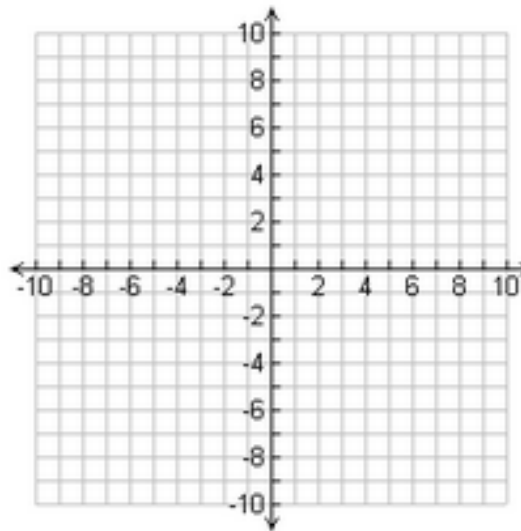
Example 6:

Graph each function and its parent function. Then describe the transformation.

(a) $g(x) = 2|x|$



(b) $h(x) = \frac{1}{2}x^2$



Example 7: For each function, write the equation of its parent function and describe its transformation.

(a) $g(x) = 3x$

(b) $h(x) = \frac{3}{2}x^2$

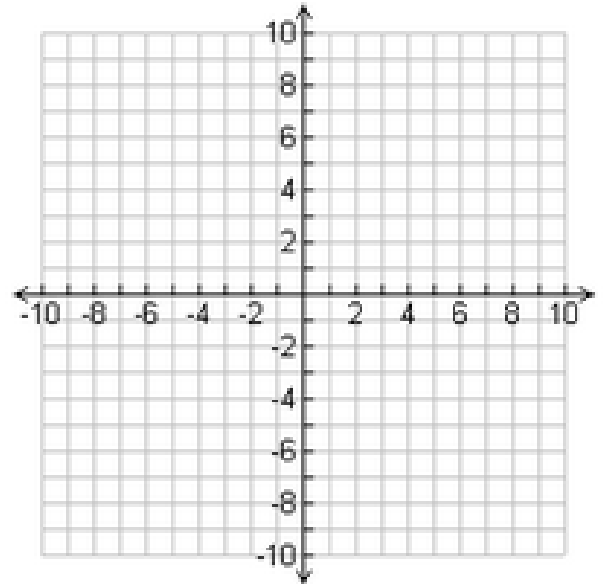
(c) $c(x) = 0.2|x|$

Combinations of Transformations

You can use more than one transformation to change the graph of a function.

Example 8:

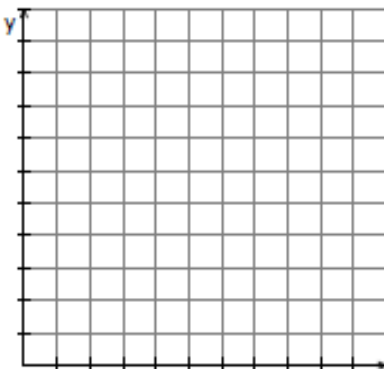
Graph $g(x) = -|x + 5| - 3$ and its parent function. Then describe the transformations.



Modeling with Mathematics

Example 9: The table shows the height y of a dirt bike x seconds after jumping off a ramp. What type of function can you use to model the data? Estimate the height after 1.75 seconds.

Time (seconds), x	Height (feet), y
0	8
0.5	20
1	24
1.5	20
2	8



Example 10: Use a graphing calculator to graph the function and its parent function. Then describe the transformation.

(a) $h(x) = -\frac{1}{4}x + 5$

(b) $d(x) = 3(x - 5)^2 - 1$

Example 11: The table shows the amount of fuel in a lawnmower over time. What type of function can you use to model the data? When will the tank be empty?

Time (minutes), x	0	10	20	30	40
Fuel remaining (fluid ounces), y	15	12	9	6	3

Example 12: Making an Argument

Your friend says two different translations of the graph of the parent linear function can result in the graph of $f(x) = x - 2$. Is your friend correct? Explain.

1.2 – Transformations of Linear and Absolute Value Functions

Essential Question: How do the graphs of $y = f(x) + k$, $y = f(x - h)$, and $y = -f(x)$ compare to the graph of the parent function f ?

What You Will Learn

- Write functions representing translations and reflections.
- Write functions representing stretches and shrinks.
- Write functions representing combinations and transformations.

Transformations of the Parent Absolute Value Function

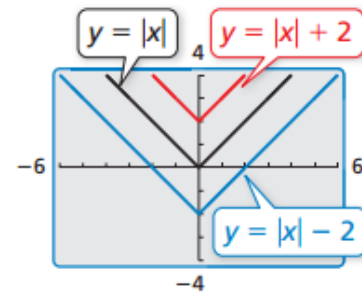
Example 1:

Compare the graph of the function

$$y = |x| + k$$

to the graph of the parent function

$$f(x) = |x|.$$



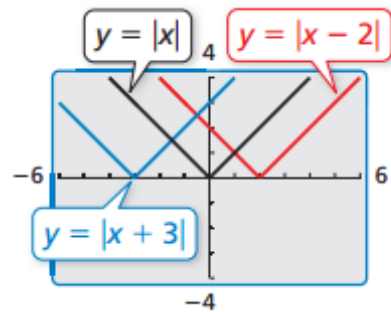
Example 2:

Compare the graph of the function

$$y = |x - h|$$

to the graph of the parent function

$$f(x) = |x|.$$



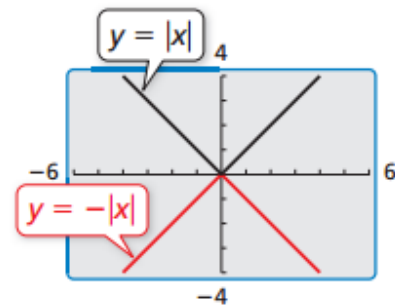
Example 3:

Compare the graph of the function

$$y = -|x|$$

to the graph of the parent function

$$f(x) = |x|.$$

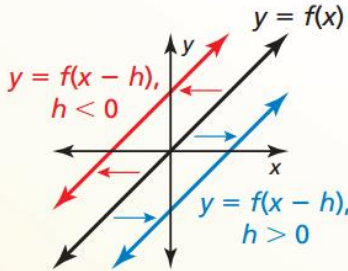


Translations

Core Concept

Horizontal Translations

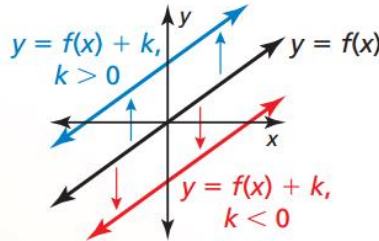
The graph of $y = f(x - h)$ is a horizontal translation of the graph of $y = f(x)$, where $h \neq 0$.



Subtracting h from the **inputs** before evaluating the function shifts the graph left when $h < 0$ and right when $h > 0$.

Vertical Translations

The graph of $y = f(x) + k$ is a vertical translation of the graph of $y = f(x)$, where $k \neq 0$.

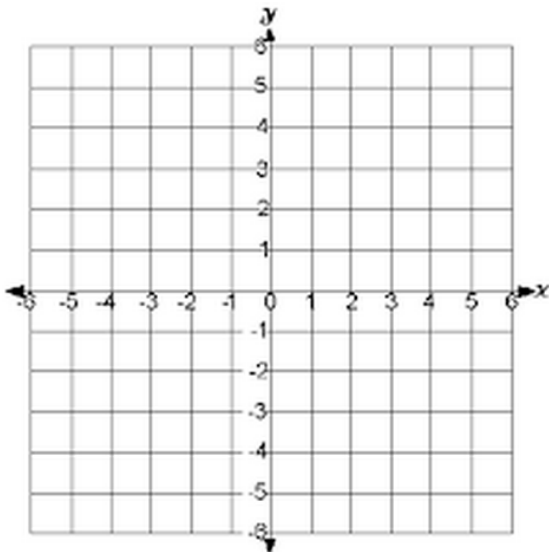


Adding k to the **outputs** shifts the graph down when $k < 0$ and up when $k > 0$.

Example 4: Writing Translations of Functions

Let $f(x) = 2x + 1$. Find the equations for $g(x)$ and $h(x)$ and graph them below.

- Write a function g whose graph is a translation 3 units down of the graph of f .
- Write a function h whose graph is a translation 2 units to the left of the graph of f .

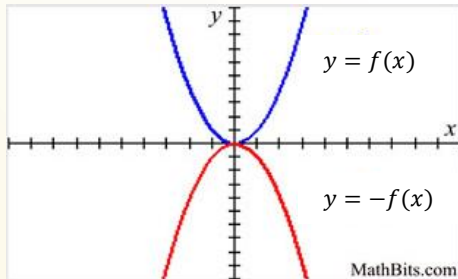


Reflections

Core Concept

Reflections in the x -axis

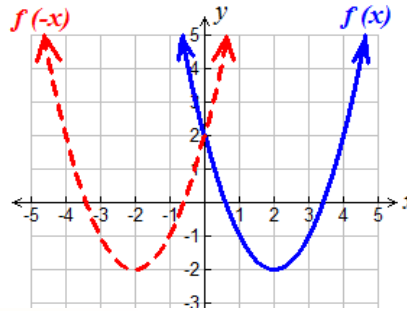
The graph of $y = -f(x)$ is a reflection in the x -axis of the graph of $y = f(x)$.



Multiplying the **outputs** by -1 changes their signs.

Reflections in the y -axis

The graph of $y = f(-x)$ is a reflection in the y -axis of the graph of $y = f(x)$.

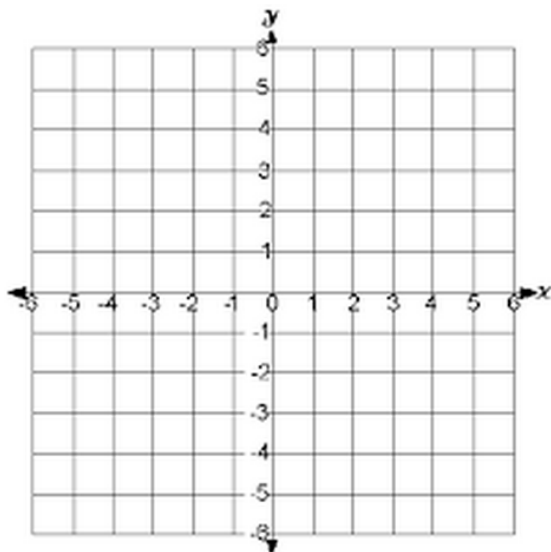


Multiplying the **inputs** by -1 changes their signs.

Example 5: Writing Reflections of Functions

Let $f(x) = |x + 3| + 1$. Find the equations for $g(x)$ and $h(x)$ and graph them below.

- Write a function g whose graph is a reflection in the x -axis of the graph of f .
- Write a function h whose graph is a reflection in the y -axis of the graph of f .



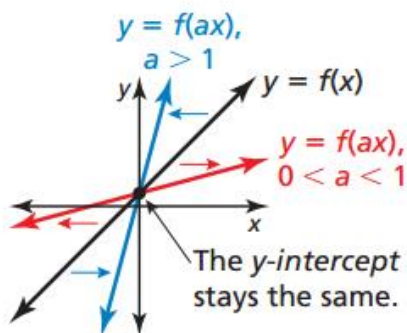
Stretches and Shrinks

Core Concept

Horizontal Stretches and Shrinks

The graph of $y = f(ax)$ is a horizontal stretch or shrink by a factor of $\frac{1}{a}$ of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.

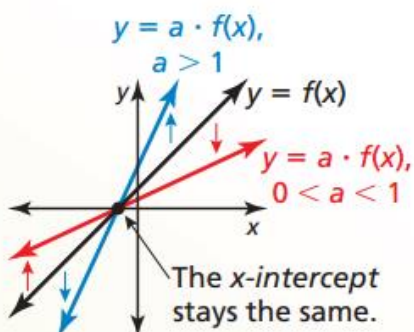
Multiplying the **inputs** by a before evaluating the function stretches the graph horizontally (away from the y -axis) when $0 < a < 1$, and shrinks the graph horizontally (toward the y -axis) when $a > 1$.



Vertical Stretches and Shrinks

The graph of $y = a \cdot f(x)$ is a vertical stretch or shrink by a factor of a of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.

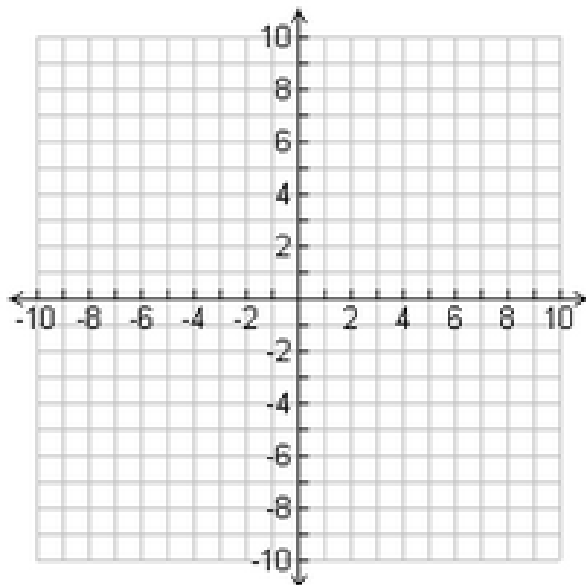
Multiplying the **outputs** by a stretches the graph vertically (away from the x -axis) when $a > 1$, and shrinks the graph vertically (toward the x -axis) when $0 < a < 1$.



Example 6: Writing Stretches and Shrinks of Functions

Let $f(x) = |x - 3| - 5$. Find the equations for $g(x)$ and $h(x)$ and graph them below.

- Write a function g whose graph is a horizontal shrink of the graph of f by a factor of $\frac{1}{3}$.
- Write a function h whose graph is a vertical stretch of the graph of f by a factor of 2.



Combinations of Transformations

You can write a function that represents a series of transformations on the graph of another function by applying the transformations one at a time in the stated order.

1. Let the graph of g be a vertical shrink by a factor of 0.25 followed by a translation 3 units up of the graph of $f(x) = x$. Write a rule for g .
2. Let the graph of g be a translation 6 units down followed by a reflection in the x -axis of the graph of $f(x) = |x|$. Write a rule for g .
3. Let the graph of g be a reflection in the y -axis followed by a horizontal stretch by a factor of 2 of the graph $f(x) = x^2$.
4. Let the graph of g a translation 3 units to the right followed by a translation 2 units up of the graph $f(x) = |x + 1|$.

Modeling with Mathematics

You design a computer game. Your revenue for x downloads is given by $f(x) = 2x$. Your profit is \$50 less than 90% of the revenue for x downloads. Describe how to transform the graph of f to model the profit. You are asked to find the profit for 100 downloads.

What if your revenue function is $f(x) = 3x$. How does this affect your profit for 100 downloads?

1.3 Modeling with Linear Functions

Essential Question: How can you use a linear function to model and analyze a real-life situation?

What You Will Learn

- Write equations of linear functions using points and slopes
- Find lines of fit and lines of best fit.

Writing Linear Equations

Core Concept

Writing an Equation of a Line

Given slope m and y -intercept b

Use slope-intercept form:

$$y = mx + b$$

Given slope m and a point (x_1, y_1)

Use point-slope form:

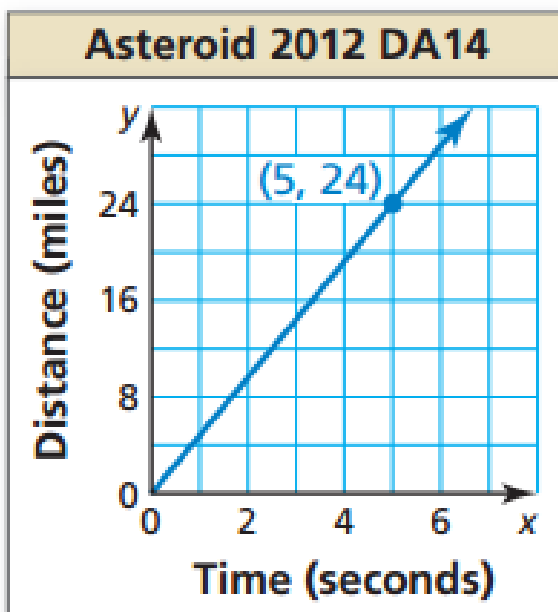
$$y - y_1 = m(x - x_1)$$

Given points (x_1, y_1) and (x_2, y_2)

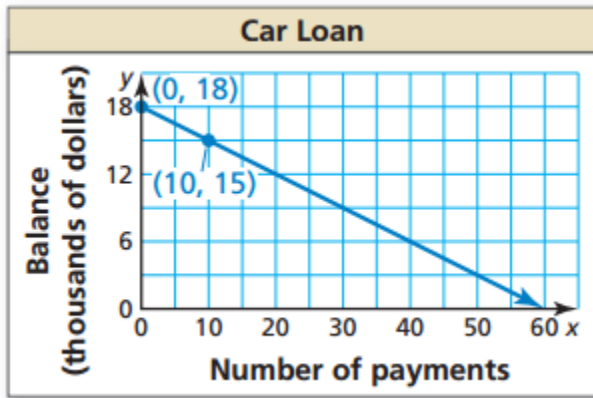
First use the slope formula to find m .
Then use point-slope form with either given point.

WRITING EQUATIONS FROM A GRAPH

Example 1: The graph shows the distance Asteroid 2012 DA14 travels in x seconds. Write an equation of the line and interpret the slope. The asteroid came within 17,200 miles of Earth in February, 2013. About how long does it take the asteroid to travel that distance?



Example 2: The graph shows the remaining balance y on a car loan after making x monthly payments. Write an equation of the line and interpret the slope and y -intercept. What is the remaining balance after 36 payments? At this point, how many monthly payments are left?



Example 3: Two prom venues charge a rental fee plus a fee per student. The table shows the total costs for different numbers of students at Lakeside Inn. The total cost y (in dollars) for x students at Sunview Resort is represented by the equation:

$$y = 10x + 600$$

Which venue charges less per student?

Lakeside Inn	
Number of students, x	Total cost, y
100	\$1500
125	\$1800
150	\$2100
175	\$2400
200	\$2700

How many students must attend for the total costs to be the same?

FINDING LINES OF FIT AND LINES OF BEST FIT

Data does not always show an *exact* linear relationship. When the data in a scatter plot shows an approximately linear relationship, you can model the data with a _____.

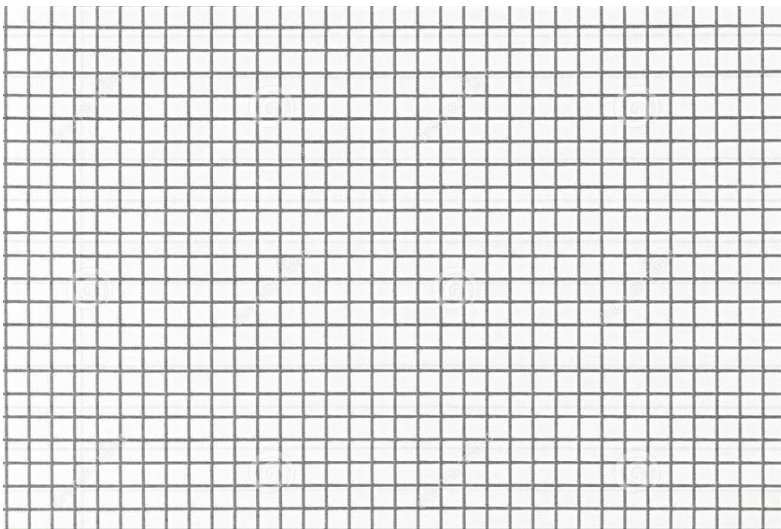
Example 4: Finding a Line of Fit

Take a look at the table and answer the following questions.

a) What does the table show?

Femur Length (in cm)	Height (in cm)
40	170
45	183
32	151
50	195
37	162
41	174
30	141
34	151
47	185
45	182

b) Graph this relationship.



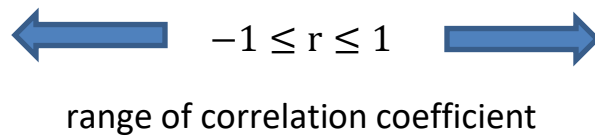
c) What type of function would best describe this relationship? _____

d) Write an equation for a line of fit.

e) Use the line of best fit to estimate the height of a person whose femur is 35 cm long.

The _____ is the line that lies as close as possible to all of the data points. Your calculators have a *linear regression* feature that can help you find the line of best fit for a set of data.

The _____, denoted by r , is a number from -1 to 1 that measures how well a line fits a set of data pairs (x, y) .



When r is near 0 , the points _____

Example 5: Using a Graphing Calculator

Use the *linear regression* feature on a graphing calculator to find an equation of the line of best fit for the data. Estimate the height of a person whose femur is 35 cm long. Compare this height to your estimate in Example 4.

MODELING LINEAR RELATIONSHIPS

Example 6: Dina was driving away from New York City at a constant speed of 58 miles per hour. She started 45 miles away.

- a) Write a linear function that gives Dina's distance, D , from New York City as a function of the number of hours, h , she has been driving.

- b) If Dina's destination is 270 miles away from New York City, algebraically determine to the nearest tenth of an hour how long it will take her to reach her destination.

1.4 - Solving Linear Systems

Essential Question: How can you determine the number of solutions of a linear system?

What You Will Learn

- Visualize solutions of systems of linear equations in three variables.
 - Solve systems of linear equations in three variables algebraically.
 - Solve real-life problems.
-

Recognizing Graphs of Linear Systems

Match each linear system with its corresponding graph. Explain your reasoning.

1. $2x - 3y = 3$

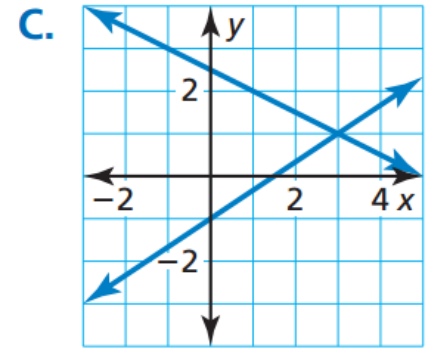
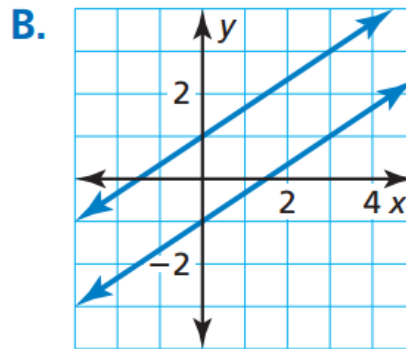
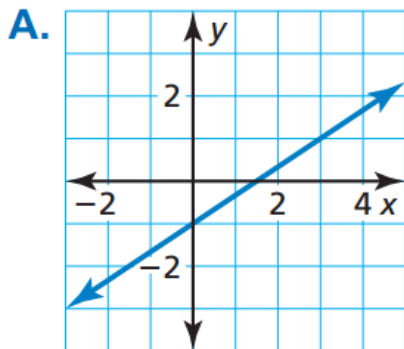
$-4x + 6y = 6$

2. $2x - 3y = 3$

$x + 2y = 5$

3. $2x - 3y = 3$

$-4x + 6y = -6$



How can you determine the number of solutions of a linear system?

Solving Systems of Linear Equations

Solve the linear system of substitution or elimination. Then use your calculator to check your solution.

$$x + 3y = 1$$

$$-x + 2y = 4$$

Suppose you were given a system of *three* linear equations in *three* variables. Explain how you would approach solving such a system.

$$x + y + z = 1$$

$$x - y - z = 3$$

$$-x - y + z = -1$$

VISUALIZING SOLUTIONS OF SYSTEMS

A **linear equation in three variables** x , y , and z is an equation of the form:

$$ax + by + cz = d \rightarrow \text{where } a, b, \text{ and } c \text{ are not all zero.}$$

The following is an example of a **system of three linear equations** in three variables.

$$3x + 4y - 8z = -3$$

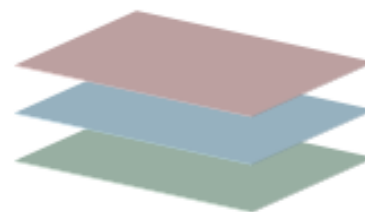
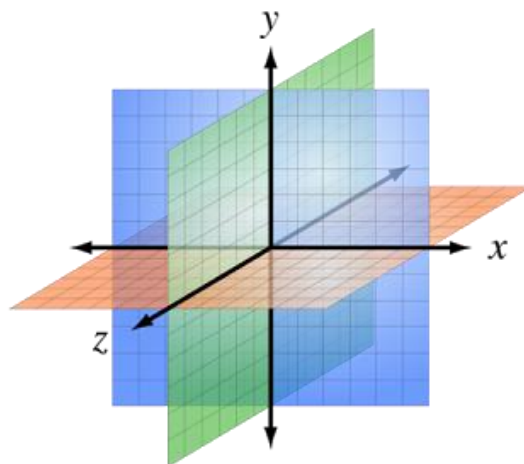
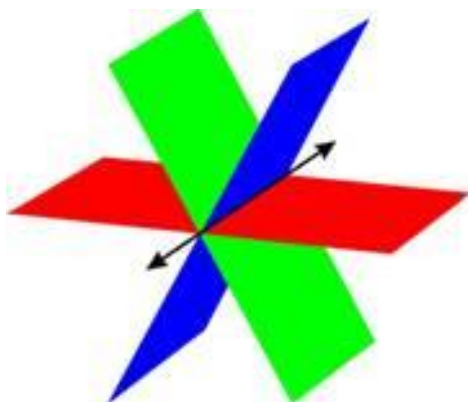
$$x + y + 5z = -12$$

$$4x - 2y + z = 10$$

A **solution** of such a system is an **ordered triple** (x, y, z) whose coordinates make each equation true.

The graph of a linear equation in three variables is a plane in three-dimensional space.

Match each linear system in three variables with a description of exactly one solution, infinitely many solutions, or no solutions. Explain.



Solving a Three Variable System

Example 1: Solve the following system.

$$4x + 2y + 3z = 12$$

$$2x - 3y + 5z = -7$$

$$6x - y + 4z = -3$$

Example 2: Solve the following system.

$$3x + 4y - 8z = -3$$

$$x + y + 5z = -12$$

$$4x - 2y + z = 10$$

Example 3: Solve the following system.

$$x + y + z = 2$$

$$5x + 5y + 5z = 3$$

$$4x + y - 3z = -6$$

Example 4: Solve the following system.

$$x - y + z = -3$$

$$x - y - z = -3$$

$$5x - 5y + z = -15$$

Solving Real-Life Problems

Example 5: An amphitheater charges \$75 for each seat in Section A, \$55 for each seat in Section B, and \$30 for each lawn seat. There are three times as many seats in Section B as in Section A. The revenue from selling all 23,000 seats is \$870,000. How many seats are in each section of the amphitheater?



Example 6: The number of left-handed people in the world is one-tenth the number of right-handed people. The percent of right-handed people is nine times the percent of left-handed people and ambidextrous people combined. What percent of people are ambidextrous?



Example 7: HOW DO YOU SEE IT?

Determine whether the system of equations that represents the circles has *no solution*, *one solution*, or *infinitely many solutions*.

